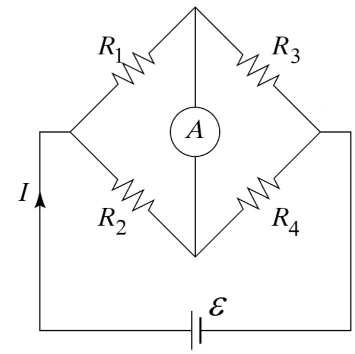
PHYS 2212
Recitation 07

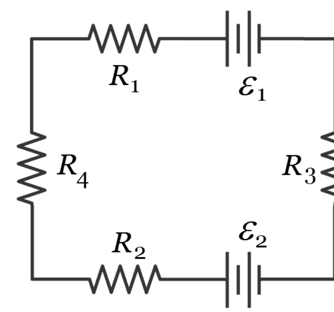
March 8-11

Example

This is a "bridge circuit". The ammeter A has resistance R_g. The ideal battery has emf $\mathcal{E}$. What is the relationship among the resistors when the current I_g through the ammeter is zero?



Problem



The batteries in the illustrated circuit are *real*. That is, emf $\mathcal{E}_1$ has internal resistance r_1, and emf $\mathcal{E}_2$ has internal resistance r_2, not shown in the diagram. $\mathcal{E}_2 > \mathcal{E}_1$, so battery 1 is being charged by battery 2.

At what rate magnitude is energy being delivered to battery 1? At what rate magnitude is energy being stored in battery 1?

PHYS 2212 **Solution**
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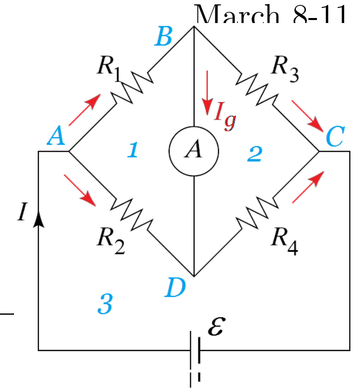

Choose current directions. Let current I_1 flow through resistor R_1, etc. Use Kirchoff's node and loop laws. Nodes are lettered A though D. Loops are numbered 1, 2, and 3.

A: $-I + I_1 + I_2 = 0$ $\quad \langle 1 \rangle$

B: $-I_1 + I_g + I_3 = 0$ $\quad \langle 2 \rangle$

D: $-I_2 - I_g + I_4 = 0$ $\quad \langle 3 \rangle$

we chose clockwise is our Direction.

we have Loop ①: $I_1R_1 + I_gR_g - I_2R_2 = 0$ $\quad \langle 4 \rangle$

lope: ②: $I_3R_3 - I_4R_4 - I_gR_g = 0$ $\quad \langle 5 \rangle$

③: $I_2R_2 + I_4R_4 = \mathcal{E}$. $\quad \langle 6 \rangle$

From those 6 equations

Now we only want to get ($I_g=0$), so we do not need to solve these six equation together.

we put $I_g=0$ in <2> <3>.

$\Rightarrow I_1=I_3,\ I_2=I_4$

in <4> $I_1R_1=I_2R_2$

in <5> $I_3R_3=I_4R_4$

$$\Rightarrow \frac{R_1}{R_3}=\frac{R_2}{R_4} \Rightarrow R_1R_4=R_2R_3$$

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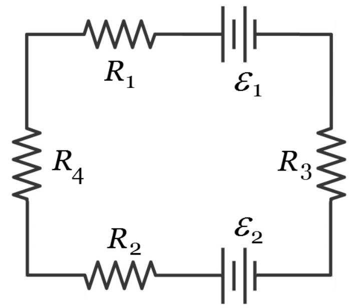

At what rate magnitude is energy being delivered to battery 1? At what rate magnitude is energy being stored in battery 1?

Remember that power is the rate of energy transfer. Use Kirchoff's Loop Law. Traverse the circuit clockwise, in the known direction of current flow.

$$-I(R_1+r_1)-I(R_2+r_2)-I_3R_3-I_4R_4-\mathcal{E}_1+\mathcal{E}_2=0$$

$$\Rightarrow \quad I=\frac{\mathcal{E}_2-\mathcal{E}_1}{R_1+R_2+R_3+R_4+r_1+r_2}$$

Total power delivered to $\mathcal{E}_1$ is

$$P = I\mathcal{E}_1 + I^2 r_1$$

$$=\left(\frac{\mathcal{E}_2-\mathcal{E}_1}{R_1+R_2+R_3+R_4+r_1+r_2}\right)\mathcal{E}_1+\left(\frac{\mathcal{E}_2-\mathcal{E}_1}{R_1+R_2+R_3+R_4+r_1+r_2}\right)^2 r_1$$

The rate that energy is stored in $\mathcal{E}_1$ is

$$P = I\mathcal{E}_1 = \left(\frac{\mathcal{E}_2-\mathcal{E}_1}{R_1+R_2+R_3+R_4+r_1+r_2}\right)\cdot\mathcal{E}_1$$

Note that if magnitudes had not been specified, both of these powers would need to be negative, as power *provided by* a circuit element is considered positive.